8.3 - Analyzing Geometric Sequences and Series

Warmup

Find the indicated sum for each series.

$$\boxed{\text{Geometric} \\ S_n = \frac{a_1(1-r^n)}{1-r}}$$

1) $1 + 2 + 4 + \ldots;\ S_8$

255

2) $3 + 6 + 12 + \ldots;\ S_7$

381

3) $4 + 12 + 36 + \ldots;\ S_6$

1456

4) $7 - 35 + 175 - \ldots;\ S_5$

3647

5) $7 + 70 + 700 + \ldots;\ S_7$

$7{,}777{,}777$

6) $11 + 33 + 99 + \ldots;\ S_8$

$36{,}080$

8.5 - Using Recursive Rules with Sequences

An **explicit rule** gives a_n as a function of the term's position number n in a sequence.

A **recursive rule** give the beginning term and a recursive equation.

Explicit Definition

$$a_n = a_1 + (n - 1)d$$

$$a_n = 5 + (n - 1)4$$

$$a_n = 4n + 1$$

Recursive Definition

$$a_1 = 5$$

$$a_n = a_{n-1} + 4$$

8.5 - Using Recursive Rules with Sequences

Recursive Equations for Arithmetic and Geometric Sequences

Arithmetic Sequence

$a_n = a_{n-1} + d$, where d is the common difference

Geometric Sequence

$a_n = r \cdot a_{n-1}$, where r is the common ratio

Write a recursive rule for the sequence.

$a.\ 2, 14, 98, 686, 4802, \ldots$

$b.\ 19, 13, 7, 1, -5, \ldots$

$a_1 = 2;\ a_n = 7a_{n-1}$

$a_1 = 19;\ a_n = a_{n-1} - 6$

8.5 - Using Recursive Rules with Sequences

Find the first 6 terms.

$a_1 = 1$

$a_2 = 1$

$a_n = a_{n-2} + 2 \cdot a_{n-1}$

$a_3 = ??$

$a_3 = a_1 + 2 \cdot a_2$

$a_3 = 1 + 2 \cdot 1$

$a_3 = 3$

$a_4 = ??$

$a_4 = a_2 + 2 \cdot a_3$

$a_4 = 1 + 2 \cdot 3$

$a_4 = 7$

$1, \ 1, \ 3, \ 7, \ 17, \ 41$

8.5 - Using Recursive Rules with Sequences

Find the first 6 terms.

$$a_1 = 1$$

$$a_2 = 1 \qquad \qquad 1, \ 1, \ 3, \ 7, \ 17, \ 41$$

$$a_n = a_{n-2} + 2 \cdot a_{n-1}$$

Practice - Find the first six terms

1) $a_1 = 8$

$\quad a_n = a_{n-1} - 1$

$\qquad$ 8, 7, 6, 5, 4, 3

2) $a_1 = 1, \ a_2 = 2$

$\quad a_n = (a_{n-2})^2 + a_{n-1}$

$\qquad$ 1, 2, 3, 7, 16, 65

8.5 - Using Recursive Rules with Sequences

Write a recursive rule for the sequence.

1
2
4
8
16 …

$a_1 = 1, \, a_n = 2 \cdot a_{n-1}$

1
1
2
3
5
8
13 …

$a_1 = 1, \, a_2 = 1, \, a_n = a_{n-1} + a_{n-2}$

8.5 - Using Recursive Rules with Sequences

Practice - Write a recursive rule for the sequence.

a) $a_n = 17 - 4n$

$a_1 = 13, \; a_n = a_{n-1} - 4$

b) $\dfrac{5}{6}, \dfrac{7}{12}, \dfrac{1}{3}, \dfrac{1}{12}, \ldots$

$a_1 = \dfrac{5}{6}, \; a_n = a_{n-1} - \dfrac{1}{4}$

8.5 - Using Recursive Rules with Sequences

Fibonacci Sequence

$$1,\ 1,\ 2,\ 3,\ 5,\ 8,\ 13,\ \ldots \qquad a_1 = 1 \qquad a_2 = 1$$

$$a_n = a_{n-2} + a_{n-1}$$

Practice - Write a recursive formula (hard)

$a)$ $1, 1, 3, 7, 17, 41, 99, \ldots$ $\qquad$ $b)$ $1, 2, 3, 7, 22, 155, \ldots$

$a_1 = 1,\ a_2 = 1$ $\qquad\qquad\qquad$ $a_1 = 1,\ a_2 = 2$

$a_n = 2 \cdot a_{n-1} + a_{n-2}$ $\qquad\qquad$ $a_n = a_{n-1} \cdot a_{n-2} + 1$

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